Three different empirical formulae were used to analyze the total reaction cross section for proton scattering on $^{6-11}\text{Li}$, $^{7-14}\text{Be}$ and $^{8-17}\text{B}$ in the energy range 40-1000 MeV. The parameters of these formulae were determined and their variation with both the atomic mass number and proton incident energies are clarified. The contribution of excess target neutrons is estimated and discussed. The black-sphere radii of the target nuclei are determined and a comparison with the available root-mean-square radii of the nuclear matter distribution is presented.

**Keywords:** Glauber and Black Sphere Models, Total Reaction Cross-Section, Proton-Nucleus Scattering.

**INTRODUCTION AND MATHEMATICAL METHOD**

Light exotic nuclei are those having unusual nucleonic composition. An exotic isotope contains many more or many fewer neutrons than a stable isotope of the same element [1]. The lifetime for this kind of nuclei, ranging from millisecond up to few seconds, is much larger than the period of nucleonic motion ($\sim 10^{-23}$ s) inside the nucleus "time scale". Therefore, they have long enough lifetimes to possess well-defined many-body structure as bound (or quasi-bound) system of nucleons [1].

Study of proton scattering from exotic light nuclei has opened an exciting channel to look for some crucial issues in context of both nuclear structure and nuclear astrophysics [1, 2]. The quantities usually measured in the studies include various inclusive cross-sections, for example: reaction or interaction, nucleon-removal, Coulomb breakup cross-sections as well as momentum distributions of fragments. These quantities have played an important role to reveal some information’s about the
nuclear properties of these nuclei. The total reaction cross-section ($\sigma_R$) is one of the most fundamental quantities characterizing the nuclear reactions and is considered as a probe for nuclear structure details [2-4].

Tag El-Din et al [5] calculated the total reaction cross-section for proton scattering on $^{6-11}$Li, $^{7-14}$Be and $^{8-17}$B isotopes, within the optical limit approximation of Glauber multiple scattering model [6-8] in the energy range from 30 to 1000 MeV. The densities of the target nuclei are described by realistic mean field approach (RMF) and its extension which is based on the field theory motivated effective Lagrangian (E-RMF) for finite nuclei. The parameters of these densities were adjusted to both the root-mean square radii of charge distributions and the binding energies of the target nuclei [9], where the difference between the density distribution of protons and neutrons are not taken into consideration. In addition, the parameters of nucleon-nucleon elastic scattering amplitude are introduced as an average of proton-proton and proton-neutron. In general, a plausible agreement with the available experimental data was found over the whole energy range [5].

On the other hand, using Glauber model [6], Abu-Ibrahim et al [10] calculated and systematically studied the total reaction cross-sections of Carbon isotopes with N = 6-16 ($^{12-22}C_{6-16}$) on a proton target for a wide range of incident energies starts from 40 MeV to 800 MeV. The intrinsic structure of the Carbon isotopes are described by a Slater determinant generated from a phenomenological mean-field potential. To go beyond the simple mean-field model, they adopted two types of dynamical models: one is a core +n model for odd-neutron nuclei, and the other is a core + n + n model for even-neutron nuclei, so, they treated the scattering of proton-proton and proton-neutron separately, i.e. the difference between both protons and neutrons distribution inside the nuclei are considered. They made a systematic analysis of $\sigma_R$ for any C-isotope from $^{12}C_6$ by proposing the following empirical formula [10],

$$\frac{\sigma_R(p^{A_{6-16}}C_6)}{\sigma_R(p^{12C_6})} = R(c) \left\{ \frac{6\Delta\sigma_{pp}^t + N \sigma_{pn}^t}{6\Delta\sigma_{pp}^t + 6\Delta\sigma_{pn}^t} \right\}, N \geq 7 \quad (1)$$

It was found that the value of R(c) is a weakly energy dependent within a certain uncertainty [10] with R(c) = 0.96 ± 0.05 over the studied energy range. $\sigma_{pp}^t$ ($\sigma_{pn}^t$) is the proton-proton (proton-neutron) total cross-section at a given energy.

The main objective of the present work is to analyze the previous calculations of the total reaction cross-section for proton scattering of $^{6-11}$Li, $^{7-14}$Be and $^{8-17}$B isotopes using three empirical formulae.
RESULTS AND DISCUSSION

1- Scale Parameter R(c) Formula

According to Eq.(1), the values of the scale parameter R(c) are calculated for proton scattering on $^{6-11}$Li, $^{7-14}$Be and $^{8-17}$B over the energy range $40 \leq E_p \leq 1000$ MeV. In these calculations, the total nuclear reaction cross sections $\sigma_R$ were considered from [5]. The values of $\sigma_{pp}$ and $\sigma_{pn}$ at the used incident proton energies are taken from the study of $\sigma_R$ for Carbon isotopes on a proton [10]. Figures (1), (2) and (3) shows the variation of R(c) with the incident proton energy for proton scattering on Li, Be and B isotopes, respectively. The values of R(c) cover 55 numerical results (5 isotopes times 11 energy points) for $^{6-11}$Li, 77 numerical results (7 isotopes times 11 energy points) for $^{7-14}$Be and 99 numerical results (9 isotopes times 11 energy points) for $^{8-17}$B. The lines which connect the data points in the figures are of order five polynomial fit. It is apparent that the values of R(c) increase with increasing proton energy up to $E_p \approx 500$ MeV, and then becomes almost constant for higher proton energies. The energy dependence is more remarkable for isotopes having higher atomic mass number. The constant value of R(c) at $E_p > 500$ MeV in the three figures approaches 0.96 especially for isotopes with higher mass number. This value agrees well with the work of Abu-Ibrahim et al. for p – Carbon isotopes [10].

![Figure 1](image-url)  
*Figure 1.* Fitting curves of the scale parameter R (c) and the incident proton energy for the Li-isotopes. The curves are arranged downwards according to the atomic mass number and E is the incident proton energy.
For p-\(^9\)Be, using the average values of \(R(c)\) at each energy, where the experimental data is available [5], it is found that the calculated values of \(\sigma_R\) are less than the experimental data by about 9%. However, applying the average value of \(R(c)\) at all considered energies, the uncertainty in \(\sigma_R\) value reached about 16%. On the other hand, introducing the average value of \(R(c)\) for \(E_p \geq 500\) MeV rises the error in
σ_R from 1.6% (at E_p > 500 MeV) up to 43% (at lower energies). Therefore, one can determine, within a certain error < 10%, the value of σ_R for proton scattering from any isotope of nuclei by the values of R(c) ≈ 0.96 especially at E_p ≥ 500 MeV. Unfortunately, this analysis can not be done for Li and B isotopes, because the experimental data is available only for p-^6^Li and p-^8^B, where these two isotopes are taken as references in our calculations of R(c).

On the other hand, Figure (4) displays the dependence of R(c) on the atomic mass number A of Li-isotopes for different proton energies (40, 100, 500, 800 and 1000 MeV). It is noticed that as the mass number increases, the values of R(c) decrease. The rate of decrease becomes more pronounced at lower proton energy.

![Figure 4](image-url)  
**Figure 4.** Fitting curves of the scale parameter R (c) and the mass number of Li-isotopes at E_p = 40, 100, 500 and 1000 MeV.

In general, it is clear that our study indicated the same manner for the dependence of R(c) on both proton energy and atomic mass number of the target isotopes for the three studied target elements. Comparing these results with those obtained by Abu-Ibrahim et al.[10], which gave a constant value of about 0.96 for R(c), regardless of the proton energy or mass number indicate that there is a major problem, specially at low energy (E_p < 500 MeV) and also at high atomic mass number. The reason of this conflict could be attributed to the different ways of calculations for the total reaction cross-section as indicated in the introduction of this work. It is clear that the distinction between the matter density distribution and the charge density distribution in the target nuclei plays an important role in the total reaction cross-section of proton interaction with the target nuclei, especially at low proton energy and at high neutron to proton ratio.
2- Neutron Excess Contribution

According to Eq.(1), the neutron contribution (NC) to the total reaction cross section $\sigma_R$ could be estimated from the ratio similar to that of [10],

$$\text{NC} \sim \frac{N \sigma_{pn}}{Z \sigma_{pp} + N \sigma_{pn}} \tag{2}$$

Our calculations for NC clarified that at certain proton energy, as the mass number of the isotope increases, the value of NC is slightly increases. For a given isotope, NC values decrease with increasing proton energy. This can be explained in terms of energy dependence of the ratio $\sigma_{pn}/\sigma_{pp}$, as it decreases with energy in the whole energy range. Abu Ibrahim et al [10] argued this point through the calculations of the reaction probabilities from proton and neutron distributions. They concluded that the neutron contribution to $\sigma_R$ is about two times of the proton contribution at 40 MeV, while at 800 MeV, the proton contribution exceeds that of the neutron. This reflects the behavior that $\sigma_{pn}$ is significantly larger than $\sigma_{pp}$ at low energy region.

For higher mass number isotopes, i.e. $^{16-17}\text{B}$, the ratio of Eq. (2) reads 0.89 and 0.69 at 40 MeV and 800 MeV, respectively. This result is compatible with the result of Abu-Ibrahim et al [10] for C-isotopes.

We calculated $\Delta\sigma_R$, where $\Delta\sigma_R = \sigma_R\left(^{\text{A}}Z_{X_{N+n}}\right) - \sigma_R\left(^{\text{A}}Z_X\right)$ at 40, 100, 500 and 800 MeV for $^7$-11Li, $^9$-17B and $^8$-14Be, where $n$ is the number of excess neutrons ($n= 1, 2, 3, \ldots$). Figs. 5, 6 and 7 reveal that the relation between $\Delta\sigma_R$ and $n$ is a straight line. Fig.(8) represents the corresponding results of $\Delta\sigma_R$ and $n$ for the reaction cross section of $p^{12-22}\text{C}$ [10] at the same energies.

Table 1. Average slopes for the relation between $\Delta\sigma_R$ and $n$. 

<table>
<thead>
<tr>
<th>Li-isotopes</th>
<th>B-isotopes</th>
<th>Be-isotopes</th>
<th>C-isotopes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>23.6</td>
<td>21.2</td>
<td>19.75</td>
<td>18.10</td>
</tr>
</tbody>
</table>
Figure 5. Fitting the straight lines of $\Delta \sigma_R$ and $n$ for Li-isotopes a 40, 100, 500 and 1000 MeV, $n$ is the excess neutrons above $N = 3$.

Figure 6. Fitting the straight lines of $\Delta \sigma_R$ and $n$ for Be-isotopes a 40, 100, 500 and 1000 MeV, $n$ is the excess neutrons above $N > 4$. 
Figure 7. Fitting the straight lines of $\Delta \sigma_R$ and $n$ for B-isotopes a 40, 100, 500 and 1000 MeV, $n$ is the excess neutrons above N>5.

Figure 8. Fitting the straight lines of $\Delta \sigma_R$ and $n$ for C-isotopes a 40, 100, 500 and 1000 MeV, $n$ is the excess neutrons above N>6.

Due to some restrictions on applying high energy approximation of GMSM at low energies, these straight lines are fitted taking into consideration the data point for $\sigma_R$ at 40MeV(case 1) and without this data point (case 2). In Table (2), the slopes of the two cases are listed, and a new formula for estimation of $\sigma_R$ for any isotope can be written as,
\[ (\sigma_R^A)_{E} = (\sigma_R^{A_1})_{E} + nS \]  \hspace{1cm} (3)

where \((\sigma_R^A)\) and \((\sigma_R^{A_1})\) are the total reaction cross-section for the nucleus of lowest atomic mass number and nuclei with higher mass number, respectively and \(S\) represents the slope. \(n\) is still the number of excess neutrons. Applying Eq.(3), it is found that the accuracy in \(\sigma_R\) is (1-4) % for \(^7\)Li, (1-9) % for \(^11\)Li, 1-7 % for \(^10\)Be, (1-10) % for \(^{14}\)Be, (2-5) % for \(^{10}\)B and (2-9) % for \(^{16}\)B with the case 2 and case 1 respectively. However, it is noticed that the accuracy became 13-20 % for \(^{16}\)C and 5-27 % for \(^{20}\)C. So, for Li, Be and B isotopes, considering the average value for all the slopes including the results at 40 MeV, i.e, \(S = 21.42 \text{ mb/neutron}\), it is apparent that the uncertainties in the values of \(\sigma_R\) didn't exceed 7 %.

3- Black–sphere Formula

It is interesting to introduce the black sphere model, where the total reaction cross-section is related to the black–sphere radius by the formula \([10, 11]\),

\[ \sigma_R = \pi a^2, \]  \hspace{1cm} (4)

where \(a\) represents the black–sphere radius. Following the Carlson’s prescription, the parameter \(a\) can be represented in terms of the mass number \(A\) by the relation \([10, 12]\)

\[ a = c_o + r_o A^{\frac{1}{3}}. \]  \hspace{1cm} (5)

This expression includes \(a A^{\frac{1}{3}}\) correction in addition to the simple geometrical \(A^{\frac{2}{3}}\) term. The parameter \(c_o\) implies the strength of \(A^{\frac{1}{3}}\) Correction to \(A^{\frac{2}{3}}\) dependence of the total reaction cross-section. In this work, the black-sphere radius \(a\), is calculated from our previous results of \(\sigma_R\) \([5]\) at all energies for the whole Li-isotopes as an example. From Eq.(5), it is found that the relation between the radius parameter \(a\) and \(A^{\frac{1}{3}}\) can be represented by straight lines at each considered energy. From the fitting of the data of the straight lines, one can extract the values of both \(r_o\) and \(c_o\) as a function of energy. Figs. (9) and (10) elucidate the dependence of \(r_o\) and \(c_o\) on proton energy. It is apparent that the total cross section can be estimated for any Li-isotope at any energy between 30 MeV and 1000 MeV with accuracy < 10%.

Moreover, empirically, within the black-sphere model, the root-mean square radius (rms) \(r_{BS}\) of the atomic nuclei can be deduced from the relation \([10]\):
Table (2) illustrates the values of $r_{BS}$ which extracted from the results of $\sigma_R$ for Li, Be and B isotopes as reported in [5], in comparison of the corresponding root-mean square radii from nucleus-nucleus collision [13-16]. One can notice that a plausible agreement is obtained. Since, Abu-Ibrahim et al.[10] from their results of $\sigma_R$ for proton-Carbon isotopes suggested that one may empirically access to the root-mean square nuclear matter radii of Carbon isotopes just by measuring $\sigma_R$ at 100 MeV.

Table 2. Radius of nuclear matter distribution " $r_m$ " and the calculated black-sphere radii.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$r_m$(fm)</th>
<th>$r_{BS}$ (fm) (present work)</th>
<th>$E_p$(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6$Li</td>
<td>2.54±0.03[13] 2.32±0.03[14]</td>
<td>2.57 2.40</td>
<td>30 40</td>
</tr>
<tr>
<td>$^7$Li</td>
<td>2.54±0.03[13] 2.33±0.02[14]</td>
<td>2.54 2.40</td>
<td>40 49</td>
</tr>
<tr>
<td>$^8$Li</td>
<td>2.57±0.03[13] 2.37±0.02[14, 15]</td>
<td>2.64 2.36*</td>
<td>40</td>
</tr>
<tr>
<td>$^9$Li</td>
<td>2.50±0.02[13] 2.32±0.02[14]</td>
<td>2.59 2.31</td>
<td>85</td>
</tr>
<tr>
<td>$^{11}$Li</td>
<td>3.36±0.24[13] 3.12±0.16[14]</td>
<td>3.16</td>
<td>30</td>
</tr>
<tr>
<td>$^7$Be</td>
<td>2.41±0.03[13] 2.31±0.02[14]</td>
<td>2.41</td>
<td>49</td>
</tr>
<tr>
<td>$^9$Be</td>
<td>$^7$Li 5.3±0.01[13] 2.38±0.01[14]</td>
<td>2.57</td>
<td>40</td>
</tr>
<tr>
<td>$^{10}$Be</td>
<td>2.43±0.02[13] 2.30±0.02[14]</td>
<td>2.35</td>
<td>85</td>
</tr>
<tr>
<td>$^{11}$Be</td>
<td>2.73±0.05[14]</td>
<td>2.74</td>
<td>49</td>
</tr>
<tr>
<td>$^{12}$Be</td>
<td>2.59±0.06[14]</td>
<td>2.54</td>
<td>85</td>
</tr>
<tr>
<td>$^9$B</td>
<td>2.38±0.04[14]</td>
<td>2.38</td>
<td>200</td>
</tr>
<tr>
<td>$^{10}$B</td>
<td>2.56±0.23[16]</td>
<td>2.56</td>
<td>200</td>
</tr>
<tr>
<td>$^{11}$B</td>
<td>2.605±0.090[16]</td>
<td>2.64</td>
<td>200</td>
</tr>
<tr>
<td>$^{12}$B</td>
<td>2.723±0.048[16]</td>
<td>2.74</td>
<td>200</td>
</tr>
<tr>
<td>$^{13}$B</td>
<td>2.746±0.048[16]</td>
<td>2.78</td>
<td>325</td>
</tr>
<tr>
<td>$^{14}$B</td>
<td>3.00±0.010[16]</td>
<td>3.01</td>
<td>500</td>
</tr>
<tr>
<td>$^{15}$B</td>
<td>2.61±0.19[16]</td>
<td>2.89</td>
<td>342.5</td>
</tr>
<tr>
<td>$^{16}$B</td>
<td>4.10±0.46[16]</td>
<td>4.09</td>
<td>49</td>
</tr>
</tbody>
</table>

* this value is the average ($r_{BS}$) at 49 and 85 MeV.
However, in this work, the proton can be considered a good probe for the rms radii of Li and Be isotopes in the energy range 30-85 MeV, and for B isotopes in the energy range around 200 MeV.

**Figure 9.** Fitting curve of $r_o$ and proton incident energy for Li-isotopes and $E$ is the incident proton energy.

**Figure 10.** Fitting curve of $c_o$ and proton incident energy for Li-isotopes and $E$ is the incident proton energy.
4- Power Law Formula

Within the framework of microscopic optical model potential, total reaction cross-section for proton scattering from some medium and heavy stable and unstable nuclei was fitted at $E_p = 0.4, 0.7$ and $1.0$ GeV using the empirical relation [17]

$$\sigma_R = \sigma_o A^\alpha,$$

(7)

where $\sigma_o$ is the reduced cross-section and $\alpha$ represents the power law exponent and $A$ stands for mass number of the largest nucleus. Both the above two parameters depend on the incident proton energy. From the calculated values of $\sigma_R$ [5] for proton scattering on Li, Be and B-Isotopes, $\sigma_o$ and $\alpha$ are obtained using the best fitting of the formula (7).

For p-Li isotopes, Figs (11) and (12) display the relation of $\sigma_o$ and $\alpha$ in connection with the proton energy. From these two figures, it was possible to estimate $\sigma_R$ for proton scattering from $^{6-11}$Li at any energy in the region $30 \leq E_p \leq 1000$ MeV, with an accuracy $\pm 5$ mb. Fig.(12) explores that $\alpha$ parameter changes from 0.65 into 0.86 in the whole energy range. This result slightly deviates from 0.644 [17]. For completeness of this section, we obtained a noticeable relation between the reduced cross-section $\sigma_o$ and the average of nucleon-nucleon total cross-section $\bar{\sigma}_t$

$$\bar{\sigma}_t = \frac{Z \sigma_{pp} + N \sigma_{pn}}{Z + N},$$

where both $\sigma_{pp}$ and $\sigma_{pn}$ are defined above. It is found that the ratio $\frac{\bar{\sigma}_t}{\sigma_o} \simeq 1.4$ for $30 \leq E_p \leq 49$ MeV and $\frac{\bar{\sigma}_t}{\sigma_o} \simeq 1.3$ in the energy range $85 \leq E_p \leq 1000$ MeV for p-Li, Be and B isotopes. Then, within error (5 – 10) %, one can replace $\sigma_o$ by $\bar{\sigma}_t$ in the energy range $E_p \geq 100$ MeV.
SUMMARY AND CONCLUSION

In the present work, we presented three empirical different formulae to analyze the total reaction cross section for proton scattering on Li, Be and B-isotopes in the
energy range from 30-1000 MeV. It is found that the parameters of these formulae depend on both the mass number of the target nuclei and the incident proton energy. In addition, the neutrons excess contribution to $\sigma$ is studied and a new expression to estimate $\sigma$ for any isotopes from the reference (stable) one is suggested. Within these formulae, $\sigma$ can be determined with uncertainties not more than 10%, therefore one can guess $\sigma$ for proton scattering on these isotopes, where no experimental data are available, in the energy range $30 < \sigma < 1000$ MeV. Moreover, using the black-sphere formula, the radii of matter distribution of the target nuclei can be estimated using proton scattering with $E_p \leq 200$ MeV.

REFERENCES

تحليل نظامي للمقطع المستعرض الكلي للتفاعل لإستطارة بروتون على نظائر الليثيوم، البريليوم والبورون

إبراهيم أحمد أحمد تاج الدين، ساميه سعيد علي حسن و أحمد حسن عزام

قسم الطبيعة النووية التجريبية، مركز البحوث النووية، هيئة الطاقة الذرية، القاهرة، رقم بريدي 12759، مصر.

قسم الرياضيات والطبيعة النظرية، مركز البحوث النووية، هيئة الطاقة الذرية، القاهرة، مصر.

ثلاث مصغ ووضعية مختلفة تم استخدامها لتحليل المقطع المستعرض الكلي للتفاعل لإستطارة بروتون على نووية الليثيوم، البريليوم والبورون. في مدى الطاقة 0.0004 تكرون فولت. تم تعين برامترات هذه المصغ وإيضاح مدى تغييرها مع كل من عدد الكتلة وطاقة البروتون الساقط. تم تقييم ومناقشة مساهمة الزيادة النيوتوربية في الأنوية الهدف على المقطع المستعرض الكلي للتفاعل. تم تعين أصفاء أقطار الأنوية الهدف من خلال نموذج القوة المعتادة ومقارنتها مع القيم المتاحة من متوسط الجذر التربيعي لأصفاء الأقطار لتوزيع المادة النووية.